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Nonlinear Electric Circuit Analysis from a Differential Geometric Point of View

Computing Operation Points for a Special Class of Electronic Circuits

The behaviour of electrical circuits can be described by a set of algebraic- and differential equations (DAE) which can be solved by numerical analysis methods. In this project geometric methods will be used to find operation points.

Especially problematic for the numerical analysis of electronic circuits are non-linear electronic devices, whose functionality is based on the feedback principle or electronic devices, whose voltage/current characteristic includes a region of negative slope (negative differential resistance).

For modeling this class of electronic circuits, it will be necessary to use differential equations with singularities.

Van-Der-Pol-Oscillator

The degenerated Van-der-Pol-Oscillator is a simple circuit consisting of a resistor and a capacitor that are connected in a circle, described by:

$$\frac{dv}{dt} = i \qquad (1)$$

$$0 = -v - i^3 + i \qquad (2)$$

with v:voltage, i:current and where the differential equation (1) characterizes the capacity and the non-linear relation (2) defines the resistance.

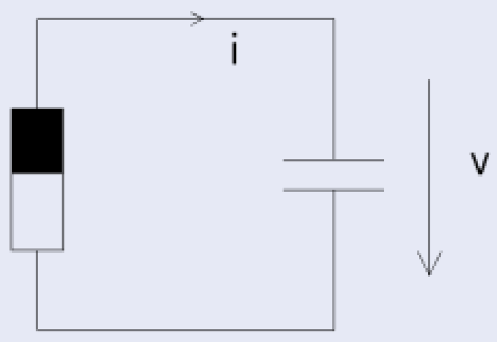

Figure 1) Cicuit of Van-der-Pol-Oscillator

A diagram of this simple oscillator is shown in Figure 1. To understand this example the shape of the curve of the nonlinear resistance is important. This is shown in Figure 2.

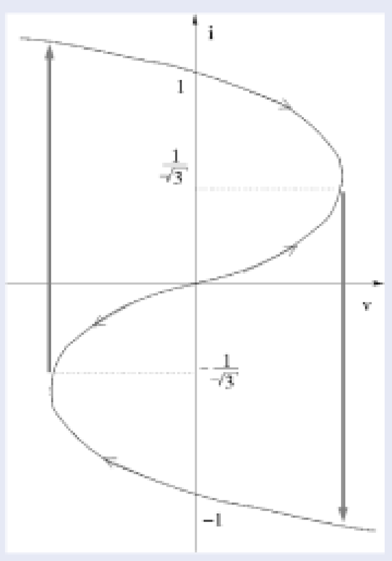

Figure 2) The resistance curve is regarded as a manifold.

The curve shown in Figure 2 may also be similarly considered as a manifold containing the dynamics. From this geometric point of view, the solution of (2) is a one-dimensional manifold, e. g., a curve M in the plane, and the differential equation generates a dynamic which should be solved with respect to the current i. But this is not feasible globally, since in the extrema of the curve wrt. v the dynamic degenerates to 0. Since $i \neq 0$ in these points, they can not be equilibria and therefore the model does not capture the behaviour of the circuit.

The described problem can be solved by a Tichonov regularisation which transforms the algebraic equation to a differential equation

$$\varepsilon\frac{di}{dt} = -v - i^3 + i. \qquad (3)$$

with ε near zero. The dynamic of the system is now generically smooth and the formerly singular points exhibit a very fast dynamic, the system "jumps" from a formerly singular point tangentially to another area of the manifold. We want to capture this phenomenon with differential geometric tools and trace the curve on the manifold to an extremum where it jumps tangentially, thus following the oscillating path.

The result of the Tichonov regularisation for the simple Van-der-Pol example is shown in Figure 3.

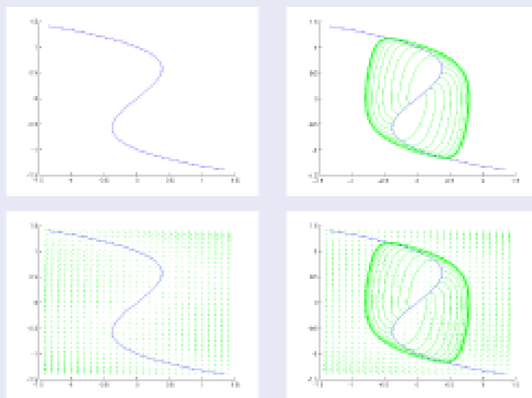

Figure 3) Result of the Tichonov regularisation for Van-der-Pol example

In higher dimensions the jumps described above occur in a special class of folded manifolds. These are embedded in a space whose axes can be associated with unlimited voltages and currents of the circuit. Now, a geometrically interpretable mapping S assigns drop points to bounce points. The drop points are located on maxima curves - the bounce points lie on a coresponding sheet of the same manifold. Such a jump can be done in different ways. (see Figure below right)

First, the points can be mapped by orthogonal projection on a resulting perpendicular curve. On the other hand, the points can be mapped by extending the current tangent vector at the time of the bounce. The last option is certainly more difficult to calculate but is still possible in an acceptable time.

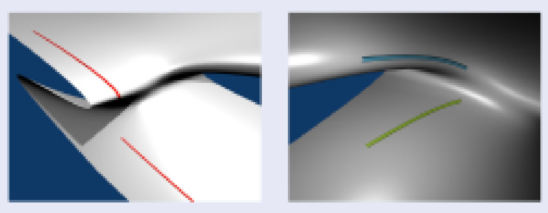

Figure 4) Tracing on Surfaces

Using Homotopy Methods for Finding Starting Points

The methods described above assume that a starting point can easily be found. This is not true for manifolds with a co-dimension greater than one. To acquire such a point it is possible to use homotopy methods which have become a powerful tool in finding solutions of various nonlinear problems, such as zeros or fixed points of maps.

A distinctive advantage of the homotopy method is that the algorithm generated by it exhibits the global convergence under weaker conditions. The homotopy concept is used to determine solutions of high-dimensional non-linear equation systems by initially finding a solution to a simpler problem and then systematically transforming it to the actual problem by embedding it in a homotopy.

Consider the non-linear equation $g(w)$=0, with 0 being a regular value. The implicit function theorem implies the existence of a curve $w(\lambda)$ which solves:

$$H(w,\lambda,w_0) = g(w) + (\lambda - 1)g(w_0), \quad \lambda \in [0,1] \qquad (4)$$

If we formulate the map $g(w)$ so that its zero set is a point on the manifold, the curve $w(\lambda)$ will converge towards it under certain conditions. This methods was used by Naß and Wolter to find solutions for similar geometric problems. By using this experience this methods can be applied to higher dimensions.

Basic Principles of Tracing Curves on Two-Dimensional Manifolds Embedded in Three-Dimensional Space

The determination of operating points of the previously described systems is still not generally solved. There exist homotopy methods that can be used to calculate the isolated zeros of the system of equations. However, these can not easily be used for oscillating systems. Moreover, for example the equations generally used in SPICE simulators are not suitable for the use of homotopy methods.

Our approach consists in the geometric interpretation of the system. We do not want to determine directly the operating points using homotopy methods. Instead, we use the homotopy-methods only to search individual starting points on the critical manifold. From there, numerical algorithms of the differential geometry are used to trace the flow on the manifold.

We can trace a curve on the manifold by numerically integrating the given differential equations which describes a tangent vector field on the manifold. This should lead us to an operation point or, if an oscillating circuit is given, represents the set of states.

Following the curve, we want to consider what happens, when the curve reaches a fold, i. e. a generalized extremum situation on the manifold. This will be the case if the circuit oscillates: the operation point jumps from the extremum to another (non-neighboring) point of the manifold. Therefore we're interested in a submanifold S_A of maximum points, as they characterize points where a jump may start. Additionally, we want to determine from the submanifold (S_A) a second one by orthogonal projection(S_B) that represents the set of points where a jump can end.

To trace a path on a two-dimensional manifold M embedded in the $\mathbb{R}^3$ we can define a differentiable manifold by

$$g(x,y,z) = 0 \qquad (5)$$

to define a path on that manifold we use the parameter t:

$$g(x(t),y(t),z(t)) = 0 \qquad (6)$$

Supposing we need the set of maximum points in z-direction, we differentiate wrt. z by:

$$g_z(x,y,z) = 0 \qquad (7)$$

Differentiate wrt. t leads to:

$$\begin{aligned} &\tfrac{d}{dt} g_z(x(t),y(t),z(t)) \\ &= (g_{zx}, g_{zy}, g_{zz}) \begin{pmatrix} x'(t) \\ y'(t) \\ z'(t) \end{pmatrix} \\ &= 0 \end{aligned} \qquad (8)$$

We can easily trace the curve of maxima if we use a function λ of x and y instead of z.

$$\begin{aligned} g(x(t),y(t),\lambda(x(t),y(t))) &= 0 \\ \Rightarrow \tfrac{d}{dt} g(x(t),y(t),\lambda(x(t),y(t))) &= 0 \\ \Rightarrow \quad (g_x, g_y, g_z)\begin{pmatrix} x' \\ y' \\ \lambda' \end{pmatrix} &= 0 \end{aligned} \qquad (9)$$

With it we are able to calculate λ.

As an example we see in the last Figures the implicit function:

$$x = -z^3 + yz^2$$

The shape of the manifold can be taken as an example set of operation points.

References

- Wolfgang Mathis, Philipp Blanke, Martin Gutschke, Franz-Erich Wolter *Analysis of Jump Behavior in Nonlinear Electronic Circuits Using Computational Geometric Methods*, Proceedings of INDS'09 2009. pp.89-95
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