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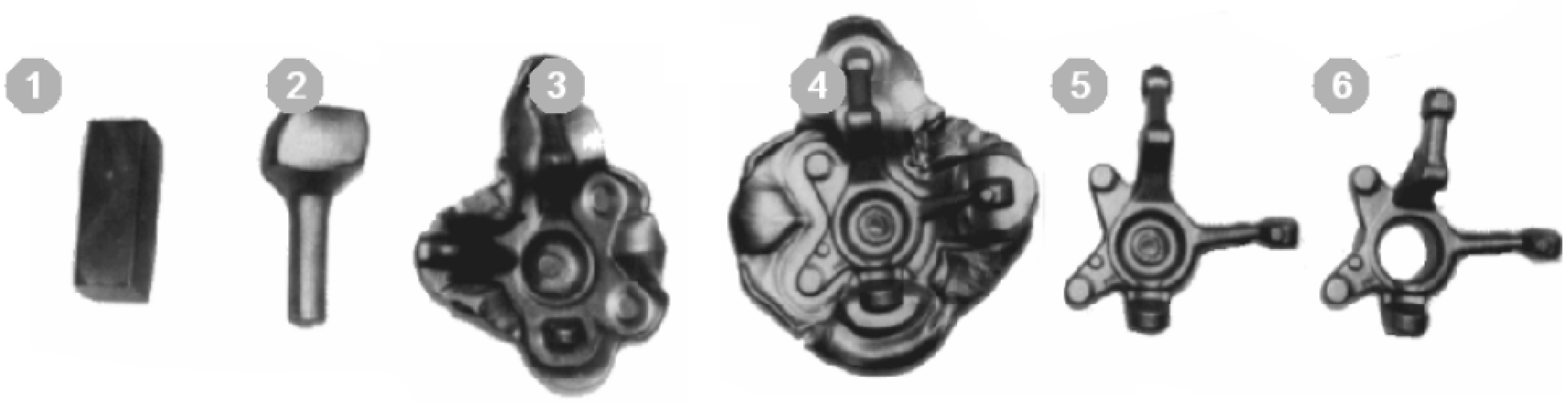
Fast Inverse Forging Simulation via Medial Axis Transform

A geometric approach towards backwards forming simulation.



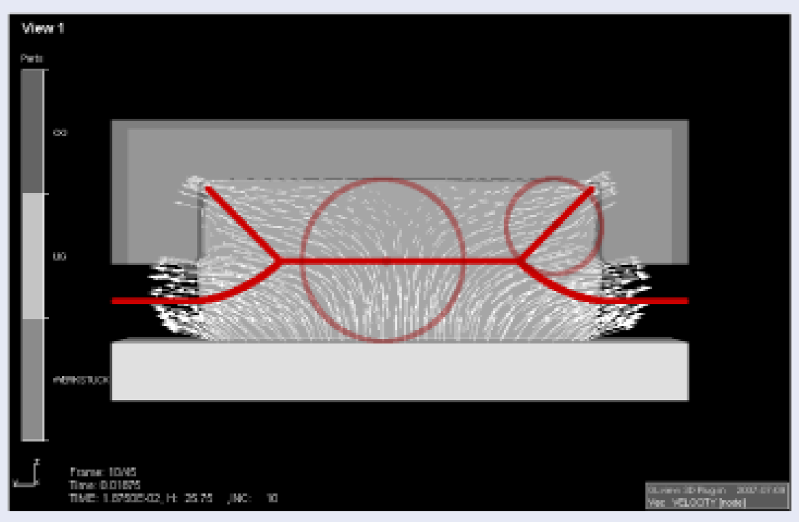
1) cutting 2) pre-bulging 3) pre-forging 4) final forging 5) chamfering 6) punching

Objective

In hot-metal drop-forging, the quality of the final product is highly dependent on the design of the forging dies and the layout of the process. Computer-aided techniques are used to reduce design time and to decrease the number of iterations until the final layout is reached. To that end, simulations are developed from equations of general plasticity theory, then Finite Element Analysis (FEA) is employed to find a solution to the so derived partial differential equations. FEA is used to verify the die designs that were accomplished by using empirical relationships or based on engineering practice. For complex parts, several steps are needed to deform the initial simple shape to the end shape with optimal properties and within a geometrical tolerance. *Conventional simulation starts with the non-deformed part and result in the final shape, while the engineer has the specifications of the final product and wants to derive the preform.* These procedures are obviously opposed.
We propose a geometric approach for a direct inverse simulation, based on the work of Mathieu et al. [2] which will help the engineer in laying out the process.

An alternative inverse simulation

As an alternative to FEM simulations, Mathieu et al. proposed an approach based on experimental observation and elementary plasticity theory [2]. In drop forging experiments, Mathieu noticed that the material flow followed specific paths. These paths are an application of the Medial Axis concept.



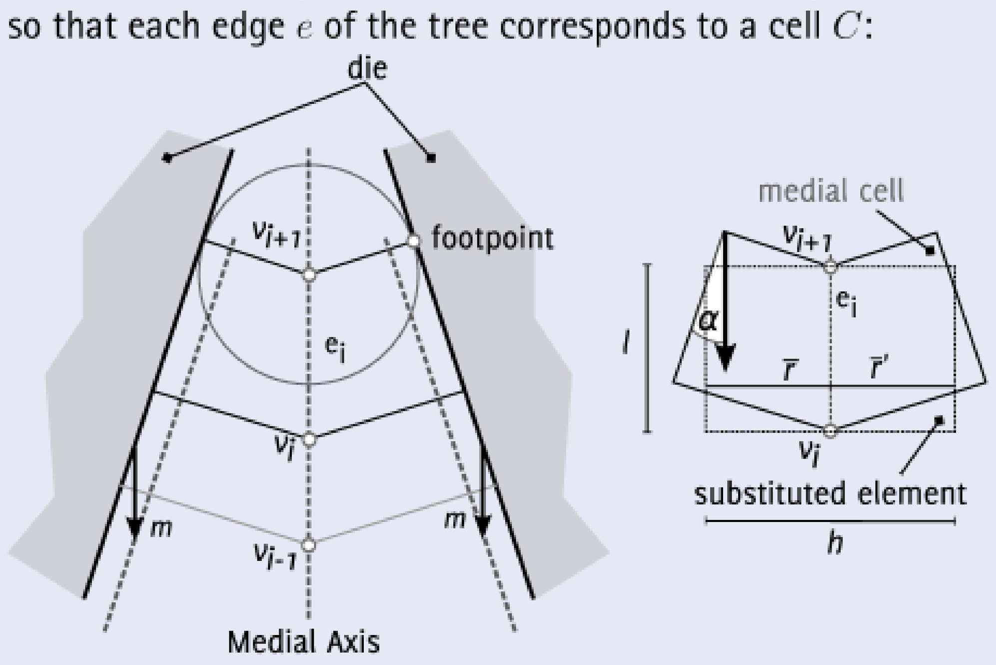
Cut through forging die with velocity field and Medial Axis of the die cavity with two maximal balls.

Definition 1 (Medial Axis Transformation). Let S be a solid in $\Re^n$. The *Medial Axis* M_S of S is defined as the closure of centers of maximal n-balls in S. The *radius function* $r : M_S \rightarrow \Re$ assigns the radius of the corresponding maximal ball to every point in M_S.

Definition 2 (Maximal Ball). An n-dimensional ball $B_{c,r} \subset S$ with center $c \in \Re^n$ and radius r is called *maximal* in S if there exists no other ball $B_{c',r'} \subset S$ that contains $B_{c,r}$.

2D-Algorithm

Input to the algorithm is the geometry of the tool, which is given by a point sample of a planar axial cut through the die surface. We assume that the material exhibits rigid-perfectly plastic behavior. We use a standard Cartesian coordinate system with orthogonal axes $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$. The deformed volume has a constant thickness in $\boldsymbol{z}$ direction that is cancelled out in the equations, and the cut will be in the $\boldsymbol{x}, \boldsymbol{y}$ plane. Furthermore, we describe the material by its flow curve together with a constant working temperature t and a constant friction coefficient μ. Finally we require the maximum speed of deformation $\dot\phi_{\text{max}}$ during the process.
The first step is the approximation of the Medial Axis via the Voronoi diagram of the point set, that produces a connected tree of Voronoi edges and vertices. The die area is partitioned, so that each edge e of the tree corresponds to a cell C:



Cell and substituted element

Then, we determine the points where the border of the material, i.e., the material front, cuts the medial axis. At these points, which we call *end points*, material will be removed to fill the volume which will be freed when the dies move. The volume movement depends on resistance along the displacement paths, therefore we compute a **forming resistance** for each cell of the partition, based on the following consideration.
Since we are only interested in the final shape of the deformed material and neglect the influence of temperature and deformation history on the process, it is sufficient to look at the movement of the object's boundary.

Forming Resistance

To compute the forming resistance of the cell C, corresponding to the edge $E := v_1 - v_0$ between vertices v_0 and v_1 with footpoints f_0 and f_1 in distances r_0 and r_1. We substitute it by a simple element as shown in figure 3. We define the height of the substituted element to be $h := \bar r + \bar r'$, so that C has the same area as the quadrangle v_0, v_1, f_1, f_0. This yields

$$\begin{aligned}\bar r &= r_0 \cos(\angle(E,(f_0-v_0)))|E| \\ &+ |f_0-f_1|\cos(\angle((f_0-f_1),(v_1-f_1))\,|v_1-f_1|\end{aligned}$$

and an equivalent expression for $\bar r'$. The width will then be $l = |E|$. Using a result from elementary plasticity theory, we describe the local deformation resistance of the cell as

$$\sigma_i = k_f \frac{|E|}{2(\bar r + \bar r')\mu}\left(e^{\frac{-4r\mu}{|E|}} - 1\right).$$

Since the friction coefficient μ is given, the only unknown is the yield stress k_f that can be determined by a simplified Hensel-Spittel law

$$k_f = \gamma e^{m_1 T}\phi^{m_2}\dot\phi^{m_3}e^{\frac{m_4}{\phi}}$$

Here, γ is a material constant, $m_1, \ldots, m_4$ are the so called regression coefficients, ϕ is the degree of deformation and $\dot\phi$ the current deformation speed. The regression coefficients can be taken from tables or simulation programs such as FORGE and the deformation speed is limited by the user-specified maximum $\dot\phi_{\text{max}}$. That leaves the computation of the local maximum deformation degree ϕ_{max} of each cell. Since the deformation will always be positive in the height and negative in the width of the cell, the deformation degree in the height is the maximal. When the movement vector of the die surface is s_D, and it forms an angle α with the vector $f_1 - f_0$, then the change of height dh will be

$$dh = s_D \sin(\alpha).$$

This yields

$$\phi_{\text{max}} = \ln\left(\frac{h + s_D\sin(\alpha)}{h}\right).$$

With this result from [1], we can compute the forming resistance of each cell.

Resistance along displacement paths

Since we assume that material will be transported along the MA, we still have to determine the volume ratios that are moved along the different branches. We postulate that the material will always move along the path of least resistance. Therefore, the total deformation resistance is added up along each branch of the MA and the displaced volume is distributed accordingly. To avoid multiple summation, the tree structure of the MA can be used to implement a backtracking algorithm, allowing fast computation of resistances for every vertex.
Finally, the distribution of material volume will be calculated iteratively over the cells, depending on the determined resistances and a prescribed rate of transport, e.g., 5% of the volume per time-step. The latter rate is a heuristic, where further research could probably provide more transparent parameters.

Outlook

The 2D algorithm can be lifted to 3D. Approximation of the MA by filtered Voronoi-diagrams is a robust and fast method for the geometric computation that is working well in 3D. The computation can be sped up by detecting the parts of the MA that don't have to be recalculated. These are obviously the points which have footpoints on the same die part and whose associated medial ball does not intersect other die parts.
The geometric forming resistance also has a counterpart in 3D based on cells of the Voronoi diagram, so that a graph-based approach like in 2D will be implemented.

References

[1] Blanke, Philipp ; Wolter, Franz-Erich: Fast Inverse Forging Simulation via Medial Axis Transform. In: *Proc. International Conference on Cyberworlds*. Los Alamitos, CA, USA : IEEE Computer Society, 2007. – ISBN 0-7695-3005-2, S. 396–403

[2] Mathieu, Hubertus: Ein Beitrag zur Auslegung der Stadienfolge beim Gesenkschmieden mit Grat. In: *Fortschritt-Berichte VDI-Reihe 2* 213 (1991)

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